Top Yukawa-Coupling Enhanced Two-Loop Corrections to the Masses of the Higgs Bosons in the MSSM with CP Violation

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Recent results for the leading $\mathcal{O}\big(\alpha_t^2\big)$ two-loop corrections to the Higgs-boson masses of the MSSM with complex parameters are presented. The strategy of the Feynman-diagrammatic, analytical calculation is illustrated. Numerical analyses show agreement with a previous result in the limit of real parameters. Furthermore, the newly available dependence on complex MSSM parameters is investigated. Mass shifts by $\approx 1\,\text{GeV}$ for different phases underline the importance of this contribution for a precise prediction of the Higgs-boson mass spectrum in the MSSM.

I. INTRODUCTION

The Higgs-like particle which has been discovered by the experiments ATLAS [1] and CMS [2] at the LHC has given rise to substantial investigations to reveal the nature of this particle as a Higgs boson responsible for electroweak symmetry breaking. Besides the presently consistent explanation of the measurements by the Standard Model Higgs boson [3], a large variety of other interpretations is possible where the Higgs particle belongs to an extended model connected to physics beyond the Standard Model. The observed particle could also be explained as a light state within a richer spectrum of scalar particles as predicted by the theoretically well motivated minimal supersymmetric Standard Model (MSSM). The Higgs sector of the MSSM consists of two complex scalar doublets leading to five physical Higgs bosons (CP-even h, H, CP-odd A, charged $H^{\pm}$) and three (would-be) Goldstone bosons. At the tree level their masses can be parametrized in terms of the A-boson mass m_A and the ratio of the two vacuum expectation values, $\tan\beta = v_2/v_1$. CP-violation is induced in the Higgs sector via loop contributions involving complex parameters from other SUSY sectors leading to mixing between h, H and A in the mass eigenstates [4].

Masses and mixings in the neutral sector are strongly affected by loop contributions. A lot of work has been invested into higher-order calculations of the mass spectrum for the case of the MSSM with real parameters [5–18] as well as complex parameters [19–23]. The largest loop contributions originate from the top Yukawa coupling h_t, or $\alpha_t = h_t^2/(4\pi)$. The class of leading two-loop Yukawa-type corrections of $\mathcal{O}\big(\alpha_t^2\big)$ has been calculated for the case of real parameters [13, 14], applying the method of the effective potential. Together with the full one-loop result [23] and the leading $\mathcal{O}(\alpha_t\alpha_s)$ terms [22], both accomplished in the Feynman-diagrammatic approach including complex parameters, it has been implemented in the public program `FeynHiggs` [7, 15, 23–25]. The calculation of the $\mathcal{O}\big(\alpha_t^2\big)$ terms for the complex version of the MSSM, also performed in the Feynman-diagrammatic approach, has been published recently [26, 27]. An outline of the calculation and sample results for the lightest Higgs-boson mass are presented in this talk.

II. HIGGS MASSES IN THE COMPLEX MSSM

The Higgs potential of the complex MSSM is given by

$$V_{\text{H}} = \frac{g_1^2 + g_2^2}{8}\left(\mathcal{H}_2^{\dagger}\mathcal{H}_2 - \mathcal{H}_1^{\dagger}\mathcal{H}_1\right)^2 + \frac{g_2^2}{2}\left|\mathcal{H}_1^{\dagger}\mathcal{H}_2\right|^2 + \sum_{i=1}^{2}\left(\tilde{m}_i^2 + |\mu|^2\right)\mathcal{H}_i^{\dagger}\mathcal{H}_i + \left(m_{12}^2\,\mathcal{H}_1 \cdot \mathcal{H}_2 + \text{h.c.}\right) \tag{1}$$

with the gauge couplings g_1, g_2, the bilinear superpotential parameter μ, and the soft-breaking parameters $\tilde{m}_i$ (real) and $m_{12}^2 \equiv b_{\mathcal{H}_1\mathcal{H}_2}\,\mu$ (complex). The Higgs doublets are conventionally written in terms of their charged and neutral components in the following way,

$$\mathcal{H}_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad \mathcal{H}_2 = \mathrm{e}^{i\xi}\begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}. \tag{2}$$

Using the notation $\Phi = (\phi_1,\ \phi_2,\ \chi_1,\ \chi_2)$, $\Phi^- = \left(\phi_1^-,\ \left(\phi_2^+\right)^\dagger\right)$, $\Phi^+ = (\Phi^-)^\dagger$, $\mathbf{M}_\Phi^{(0)} = \begin{pmatrix} \mathbf{M}_\phi & \mathbf{M}_{\phi\chi} \\ \mathbf{M}_{\phi\chi}^\dagger & \mathbf{M}_\chi \end{pmatrix}$, the Higgs potential can be written as a polynomial in the field components,

$$V_{\mathrm{H}} = -T_{\phi_1}\,\phi_1 - T_{\phi_2}\,\phi_2 - T_{\chi_1}\,\chi_1 - T_{\chi_2}\,\chi_2 + \frac{1}{2}\Phi\,\mathbf{M}_\Phi^{(0)}\,\Phi^\dagger + \Phi^-\,\mathbf{M}_{\Phi\pm}^{(0)}\,\Phi^+ + \ldots\,, \tag{3}$$

At the tree-level, the phases ξ and $\arg m_{12}^2$ can be chosen as zero, and the *CP*-mixing entries $\mathbf{M}_{\phi\chi}$ vanish. Explicit expressions for the tadpole coefficients T_i and for the mass matrices $\mathbf{M}^{(0)}$ are listed in Refs. [23, 27].

The neutral Higgs-boson masses are derived from the poles of the propagator matrix Δ_Φ,

$$\Delta_\Phi(p^2) = -\left[\Gamma_\Phi(p^2)\right]^{-1}, \quad \text{with} \quad \Gamma_\Phi(p^2) = \left[p^2\mathbf{1} - \mathbf{M}_\Phi\right]. \tag{4}$$

The irreducible two-point vertex functions Γ_Φ contain the mass matrix $\mathbf{M}_\Phi$, which at lowest order is given by the constant matrix $\mathbf{M}_\Phi^{(0)}$ in Eq. (3). At higher orders, $\mathbf{M}_\Phi$ is shifted by the momentum dependent matrix of the renormalized self-energies $\hat{\mathbf{\Sigma}}_\Phi$ according to

$$\mathbf{M}_\Phi \quad \to \quad \mathbf{M}_\Phi^{(0)} - \hat{\mathbf{\Sigma}}_\Phi(p^2)\,. \tag{5}$$

The self-energies contain in general mixing of all fields with equal quantum numbers.

In our case, we evaluate the momentum-dependent neutral "mass matrix" in the basis of tree-level mass eigenstates (*i.e.* $\{\phi_1, \phi_2, \chi_1, \chi_2\} \to \{h, H, A, G\}$) perturbatively up to the two-loop level,

$$\mathbf{M}_{hHAG}(p^2) = \mathbf{M}_{hHAG}^{(0)} - \hat{\mathbf{\Sigma}}_{hHAG}^{(1)}(p^2) - \hat{\mathbf{\Sigma}}_{hHAG}^{(2)}(0)\,. \tag{6}$$

Therein, $\hat{\mathbf{\Sigma}}_{hHAG}^{(k)}$ denotes the matrix of the renormalized diagonal and non-diagonal self-energies for the h, H, A, G fields at loop order k. For the complex MSSM, the one-loop self-energies are completely known [23]. The leading two-loop $\mathcal{O}(\alpha_t\alpha_s)$ contributions have been obtained in the approximation of zero external momentum [22], and the same approximation has been applied to derive the leading Yukawa contributions of $\mathcal{O}(\alpha_t^2)$, described in detail in Ref. [27], extending the on-shell renormalization scheme of Ref. [23] to the two-loop level.

The masses of the three neutral Higgs bosons including the new $\mathcal{O}(\alpha_t^2)$ contributions are given by the real parts of the poles of the hHA-propagator matrix, obtained as the zeroes of the determinant of the renormalized two-point vertex function, *i.e.* solving

$$\det\left[p^2\mathbf{1} - \mathbf{M}_{hHA}(p^2)\right] = 0\,, \tag{7}$$

with the corresponding (3×3)-submatrix of Eq. (6). Mixing with the Goldstone boson yields subleading two-loop contributions; also Goldstone–Z mixing occurs in principle, which is related to the other Goldstone mixings by Slavnov–Taylor identities [28, 29] and of subleading type as well [30]. However, mixing with Goldstone bosons has to be taken into account inside the loop diagrams and for a consistent renormalization.

The renormalized self-energies in Eq. (6) require counterterms up to second order in the loop expansion. A detailed study of all required counterterms as well as explicit expressions for the renormalization constants are given in Ref. [27].

III.   NUMERICAL RESULTS

In this section numerical analyses for the masses of the neutral Higgs bosons derived from Eq. (7) are presented. The complete one-loop result including the dependence on the external momentum, and the $\mathcal{O}(\alpha_t\alpha_s)$ terms are obtained from `FeynHiggs`, while the $\mathcal{O}(\alpha_t^2)$ terms are computed by means of the corresponding two-loop self-energies specified in the previous section. The combination of all contributions is carried out according to Eq. (6) within `FeynHiggs`.

The SM parameters are put together in Tab. I, as well as those MSSM parameters that are kept for the following analyses. The residual input parameters of the MSSM are shown in the figures or their captions. The parameters μ, t_β and the Higgs field-renormalization constants are defined in the $\overline{\mathrm{DR}}$ scheme at the scale m_t.

TABLE I: Default input values of the MSSM and SM parameters.

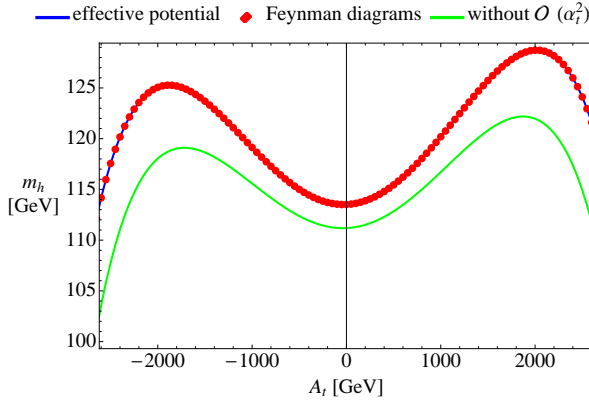| MSSM input	SM input
$M_2 = 200$ GeV,	$m_t = 173.2$ GeV,
$M_1 = \left(5 s_{\rm w}^2\right)/\left(3 c_{\rm w}^2\right) M_2$,	$m_b = 4.2$ GeV,
$m_{\tilde{l}_1} = m_{\tilde{e}_{\rm R}} = 2000$ GeV,	$m_\tau = 1.77703$ GeV,
$m_{\tilde{q}_1} = m_{\tilde{u}_{\rm R}} = m_{\tilde{d}_{\rm R}} = 2000$ GeV,	$M_W = 80.385$ GeV,
$A_u = A_d = A_e = 0$ GeV,	$M_Z = 91.1876$ GeV,
$m_{\tilde{l}_2} = m_{\tilde{\mu}_{\rm R}} = 2000$ GeV,	$G_{\rm F} = 1.16639 \cdot 10^{-5}$,
$m_{\tilde{q}_2} = m_{\tilde{c}_{\rm R}} = m_{\tilde{s}_{\rm R}} = 2000$ GeV,	$\alpha_s = 0.118$.
$A_c = A_s = A_\mu = 0$ GeV,	

FIG. 1: Comparison of the lightest Higgs-boson mass in the effective potential approach (blue) and the Feynman-diagrammatic approach (red). The curves are lying on top of each other, indicating the agreement of both calculations in the limit of real parameters. For reference the result without the contributions of $\mathcal{O}\left(\alpha_t^2\right)$ is shown (yellow). The input parameters are $m_A = 800$ GeV, $\mu = 200$, $t_\beta = 30$, $m_{\tilde{q}_3} = m_{\tilde{t}_{\rm R}} = m_{\tilde{b}_{\rm R}} = 1000$ GeV, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_{\rm R}} = 1000$ GeV, $m_{\tilde{g}} = 1500$ GeV, $A_t = A_b = A_\tau$.
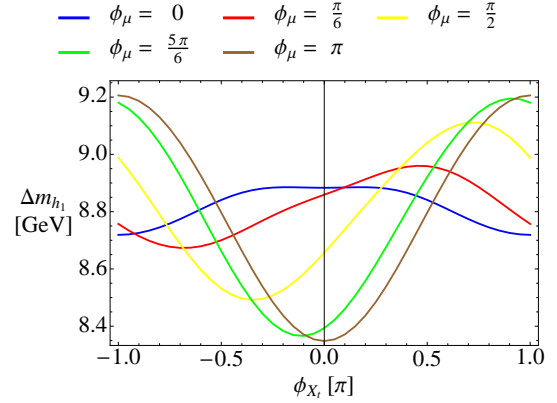
FIG. 2: The shift of the lightest Higgs-boson mass by the phases ϕ_{X_t} and ϕ_μ with respect to the case of zero phases, i.e. $\Delta m_{h_1} = m_{h_1}\left(\phi_{X_t}, \phi_\mu\right) - m_{h_1}\left(\phi_{X_t} = 0, \phi_\mu = 0\right)$. The input parameters are $m_{H^\pm} = 200$ GeV, $|\mu| = 2500$ GeV, $t_\beta = 10$, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_{\rm R}} = 1000$ GeV, $m_{\tilde{q}_3} = m_{\tilde{t}_{\rm R}} = m_{\tilde{b}_{\rm R}} = 1500$ GeV, $m_{\tilde{g}} = 2000$ GeV, $|X_t| = 2\, m_{\tilde{q}_3}$, $A_b = A_\tau = 0$.

A comparison of the obtained result in the real MSSM with the previously known $\mathcal{O}\left(\alpha_t^2\right)$ contributions [14] from a calculation making use of the effective-potential method for the mass of the lightest Higgs boson has been presented recently in Ref. [26]. An example which shows very good agreement is depicted in Fig. 1.

The phase depending contribution to the lightest Higgs mass Δm_{h_1} is illustrated in Fig. 2 showing a possible mass shift of ≈ 1 GeV for different choices of ϕ_{X_t} (with $X_t = A_t^* - \mu/t_\beta$) and ϕ_μ.

IV. CONCLUSIONS

We have presented the leading $\mathcal{O}\left(\alpha_t^2\right)$ corrections to the Higgs-boson masses of the MSSM with complex parameters. In the limit of real parameters a previous result is confirmed. Combining the new terms with the existing one-loop result and leading two-loop terms of $\mathcal{O}\left(\alpha_t \alpha_s\right)$ yields an improved prediction for the Higgs-boson mass spectrum also for complex parameters that is equivalent in accuracy to that of the real MSSM.

The numerical discussion illustrates that the mass shifts originating from the $\mathcal{O}\left(\alpha_t^2\right)$ terms are significant, and hence an adequate treatment also for complex parameters is an obvious requirement. Besides the mass shift of approximately 5 GeV in the real MSSM a further shift of ≈ 1 GeV can be induced by complex parameters.

The new terms will be included in the publicly available code `FeynHiggs`.

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